

## BOOK REVIEW

**Prandtl's Essentials of Fluid Mechanics, Second Edition.** Edited by HERBERT OERTEL, with contributions by M. BÖHLE, D. ETLING, U. MÜLLER, K. R. SREENIVASAN, U. RIEDEL & J. WARNATZ. Translated by KATHERINE MAYES. Springer, 2004. 723 pp. ISBN 0 387 40437 6. £61.50.

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Ludwig Prandtl (1875–1953) was, by any measure, one of the greatest pioneers of 20th-century fluid dynamics. As J. H. Lienhard writes in the *Dictionary of Scientific Biography* (ed. C. C. Gillespie, vol. 11, Charles Scribners & Sons, New York, 1970), “Prandtl was the founder of boundary layer theory and the originator of the German school of aerodynamics. His own work and that of his many students over half a century made Göttingen University the source of most of the elements of modern fluid mechanics. The notion of the boundary layer, the role of bound vortices in airfoil theory, the foundations of supersonic aerodynamics, explanations of drag coefficients and friction factors for a host of situations, the concept and applications of the turbulent mixing length, and the theory of wakes were results of his work.” Prandtl’s seminal paper on boundary layer theory was delivered exactly 100 years ago. This is therefore an opportune moment for the publication of this updated edition of his famous didactic text.

The first Edition of Prandtl’s *Führer durch die Strömungslehre* was published by Vieweg und Sohn, Brunswick, in 1942; the third edition appeared in 1949 and the “Authorized Translation” of this edition into English was published by Blackie & Son in 1952 under the title *Essentials of Fluid Dynamics* [sic]. Prandtl himself, at the age of 76, wrote the Preface to this volume, and explained how it had emerged from earlier works dating back to 1913. It was evidently the product of a lifetime of thought devoted to fluid dynamics and, as indicated in the subtitle, to “applications in hydraulics, aeronautics, meteorology and other subjects”. Prandtl states that the translation was the “work of Miss W. M. Deans”, who presumably would have been able to check any difficulties of scientific interpretation directly with him; certainly his Preface gives this English edition of the work his personal stamp of approval. I was lucky enough to buy my own second-hand copy in 1967 for the sum of 37/6d (£1.87 in today’s currency); it has served me well.

The volume under review is a translation of a work published in 2001 in German as “*Herbert Oertel (Hsrg.): Führer durch die Strömungslehre. 10. Auflage (10th Edition)*”. Its preface recapitulates the history of the early versions of Prandtl’s work, and states that “when the ninth edition went out of print and a new edition was desired by the publishers, we were glad to take on the task†. The first four chapters of this book keep to the path marked out by Prandtl in the first edition, in 1942. The original historical text has been linguistically revised, and leads, after the *Introduction* to chapters on *Properties of Liquids and Gases*, *Kinematics of Flow*, and *Dynamics of Fluid Flow*. . . . We have retained much of Prandtl’s original material in these chapters, . . .”. Various sections are included on more specialised topics (e.g. *Topology*

† The use of the term ‘Second Edition’ for the volume under review is consequently a bit mysterious.

of a Flow and *Flows of Non-Newtonian Media* in Chapter 3); Prandtl's original sections on *Wing Aerodynamics*, *Heat Transfer*, *Stratified Flows*, *Turbulent Flows*, *Multiphase Flows*, *Flows in the Atmosphere and Ocean*, and *Thermal Turbomachinery* have been considerably updated and extended; and three entirely new chapters on *Fluid-Mechanical Instabilities*, *Reactive Flows* and *Biofluid Mechanics of Blood Circulation* have been added.

It will be evident then that this is an ambitious work, which ostensibly seeks both to retain the spirit of Prandtl's classic treatment, and at the same time to give a contemporary flavour through inclusion of topics and themes that have been greatly developed over the last half-century. This is an extremely demanding task – it might be compared to the task of ‘updating’ the Authorized Version of the Bible! – and to my mind, it has been at best only partially successful.

There is first the problem of re-translating from Prandtl's original German, and this time without the possibility of checking directly with him over potentially troublesome points of interpretation. Consider for example the following sentence which appears in the opening paragraph of the 1952 authorized translation: “In addition to ordinary mobile liquids, there also exist very viscous liquids, whose resistance to deformation is very considerable (but vanishes when the liquid is at rest)”. In the new 2004 translation, this becomes: “As well as the usual liquids that are easy to move, there are also very viscous liquids whose resistance to change of shape is considerable, but which vanishes again at rest”. The change of wording and of punctuation is slight, but subtly damaging to the intended meaning; the 1952 version is clearly preferable. Examples of this kind abound. Here are a few more from the early sections (in each case, I give the 1952 version first, followed by the 2004 version): (i) “A liquid in equilibrium offers no resistance to change of shape” becomes “In a liquid in equilibrium, all resistance to change of shape is equal to zero”; (ii) “Water, however, is not absolutely incompressible; under high pressure it can be compressed to a noticeable extent” becomes “However, water is not fully incompressible. At high pressures it can be pressed together by noticeable amounts”; (iii) “Here we may merely mention that the ‘stress’, which represents the totality of stress vectors for all possible planes of section through a point, may be related to an ellipsoid; the stress components can accordingly be given in the form of a ‘tensor’” becomes “We point out that the *state of stress*, which represents the whole of the stress vectors in all possible cut directions through a point, can be related to an ellipsoid, and is therefore a tensor”. In each case, to my mind, Prandtl's meaning is clearer and more succinctly stated, in the 1952 version than in the 2004 version. This impression persists throughout the first four chapters of the 2004 version, here presented as a “linguistically revised” version of the original.

It may seem that I fuss unnecessarily about such points of detail; but it is the gradual accumulation of these that is liable to puzzle, and indeed irritate, as one progresses through these early chapters. Another source of irritation is the curious adoption of the ‘dot product’ notation to indicate simple multiplication. Thus for example,  $2\pi$  in Prandtl's original becomes  $2 \cdot \pi$  here,  $dp = g\rho dh$  becomes  $dp = g \cdot \rho \cdot dh$ , and so on. The convention, systematically pursued (except, significantly, in Chapter 7, *Turbulent Flows* by K. R. Sreenivasan) leads to such constructions as the following: on p. 123,  $6 \cdot \pi \cdot \mu \cdot v \cdot R = \frac{4 \cdot \pi}{3} \cdot (\rho_t - \rho) \cdot g \cdot R^3$ ; on p. 218,  $\partial\rho/\partial t + \nabla \cdot \rho \cdot \mathbf{v}$ ; on p. 400,  $\exp(i \cdot a \cdot x + i \cdot b \cdot y - i \cdot \omega \cdot t)$ ; and so on. Quite dotty, in more senses than one!

But what of the scientific content? Despite the linguistic adjustments, Prandtl's originality and insight still glimmers through. But the approach he adopted in the years before 1942, and even up to 1952, was far removed from the approach to fluid

mechanics adopted in most engineering (or applied mathematics) schools nowadays. He deliberately eschewed “complex mathematical analysis”, focussing rather on the development of “clear, intuitive apprehension”, or as we would now say, of physical insight. Where equations were unavoidable, they were frequently relegated to paragraphs in small type, a device not employed in the new edition. Thus, for example, the Navier–Stokes equations for an incompressible fluid (in small type in the 1952 edition) appear in normal type here (Chapter 4, p. 119), a promotion that they certainly merit. There is now however some unnecessary duplication, because the Navier–Stokes equations for the more general case of compressible fluids are derived in a new Chapter 5 entitled the *Fundamental Equations of Fluid Dynamics*; here, laborious Cartesian notation is used, leading to page after page of equations that provide a barrier to physical insight and even, arguably, offence to the spirit of Prandtl. I looked in vain in this chapter for any recognition of the existence of bulk (as opposed to shear) viscosity; it does not appear because the author fails to distinguish between dynamic pressure (i.e. mean normal inward stress), and pressure as defined in equilibrium thermodynamics (a distinction first clarified by Stokes himself in 1844). Other complications, such as the temperature-dependence of viscosity and thermal diffusivity, are however discussed in some detail.

As regards the completely new chapters on *Fluid-Mechanical Instabilities* (Chapter 8) and *Biofluid Mechanics of Blood Circulation* (Chapter 13) by H. Oertel, and *Reactive Flows* (Chapter 11) by J. Warnatz & U. Reidel, much material of general interest is included. Stability of flow is of course a central topic of fluid mechanics, usually treated as a preliminary to the treatment of turbulence; here the natural order is reversed, *Turbulent Flows* having been treated in Chapter 7. Chapter 8 starts with some generalities concerning the meaning of stability, and then progresses through a discussion of Rayleigh–Bénard convection, Marangoni convection, double-diffusive instability, Taylor and Görtler vortices, and shear-flow instabilities. I was troubled by a number of points; let me give just four examples (many more could be given). (i) The boundary conditions for Rayleigh–Bénard convection between two free isothermal surfaces are given (on p. 371) as the vanishing of the temperature perturbation and of the normal velocity, but no mention is made of the vanishing of tangential stress (also it is not explained why the normal velocity should vanish on a ‘free’ boundary). The student reader would find it impossible to progress from the two conditions given in equations (8.13) and (8.15) to the three given in (8.25). (ii) Following a preliminary discussion of the Rayleigh–Bénard problem, the author writes (p. 359) “Another thermal cellular convection with very similar appearance is seen in a ring gap formed by two concentric cylinders and filled with fluid. The outer cylinder is at rest, and the inner cylinder allowed to exceed a critical rotational speed.” This is of course centrifugal (Taylor) instability; there is, to be sure, a certain analogy with the Rayleigh–Bénard problem, but to describe this as “another thermal cellular convection” is quite wrong. (iii) Following a derivation of the dispersion relation for the Kelvin–Helmholtz instability of a vortex sheet at an interface between two fluids of different densities, the author writes (p. 422) “The Kelvin–Helmholtz instability is caused by the destabilizing effect of the friction that is superimposed on the stabilizing effect of the density layering.” It is of course an important feature of K-H instability that the destabilising mechanism is inviscid in character, associated with the presence of a point of inflection in the velocity profile, and it is quite wrong to attribute it to “friction”. Viscosity does play a crucial (and subtle) role in destabilising wall-bounded flows (e.g. Poiseuille flow in a channel), but that is another matter which is not discussed. (iv) At the beginning of the section on *Shear-Flow Instabilities*

(p. 395), the author makes the bland statement that “The instability of *pipe flow* occurs with three-dimensional perturbations at the critical Reynolds number  $Re_{D,crit} = 2300$ ”. This is in fact known to be a linearly stable flow for which, by careful control of perturbations, laminar flow can be maintained up to Reynolds numbers of order  $10^5$  or greater. So again, the text here is seriously misleading.

The chapters on *Turbulent Flows*, *Reactive Flows* and *Biofluid Mechanics of Blood Circulation*, are written with greater authority, and do not suffer similar defects. However, one has to take the book in its entirety, and the cumulative effect of many imperfections of the kind I have described inevitably undermines one’s confidence in it. I approached the book with high expectations, given its title which reverberates with historical associations; but I lay it down with a sense of disappointment. If I have to consult Prandtl’s *Essentials* in future, it is to my well-thumbed 1952 edition that I shall return.

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